PG – 783

I Semester M.C.A. Degree Examination, January 2015
(CBCS)
COMPUTER SCIENCE
MCA - 104 T : Discrete Mathematics

Time : 3 Hours | Max. Marks : 70

***Instruction** : Answer **any five** questions from Part – **A** and **four** questions from Part – **B**.*

PART – A (5×6=30)

1. a) Prove that "null set is a subset of every set". (3+3)

   b) For any three sets, A, B, C prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

2. a) Define :
   i) Reflexive relation
   ii) Irreflexive relation
   iii) Symmetric relation,
   iv) Antisymmetric relation and
   v) Transitive relation with an example. 6

3. Given p and q as statements, explain the following terms.
   i) Conjunction
   ii) Disjunction
   iii) Implication
   iv) Logically equivalence
   v) Tautology
   vi) Contradiction

4. If $F_0, F_1, F_2, \ldots$ are Fibonacci numbers prove that $\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}$ for all positive integers n. 6

5. An integer is selected at random from 3 through 15 inclusive. If A is the event that a number divisible by 3 is chosen and B is the event that the number exceeds 10, determine $P_r(A)$, $P_r(B)$, $P_r(A \cap B)$ and $P_r(A \cup B)$. 6

6. Prove that the open interval (0, 1) is not a countable set. 6

7. Prove that every set of 37 positive integers contain at least two integers that leave the same remainders upon division by 36. 6

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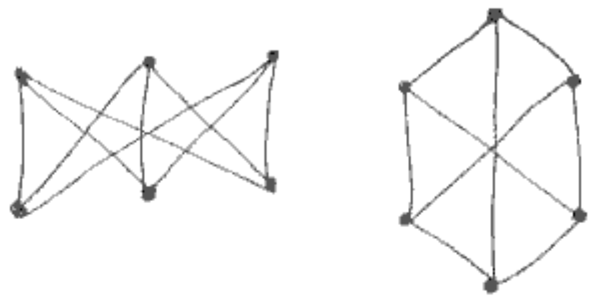
8. Define Isomorphism of graphs. Verify that the two graphs shown below are isomorphic. 6



PART – B

Answer **any four** questions. **(4×10=40)**

9. a) Using Venn diagram prove that $A\Delta(B\Delta C)=(A\Delta B)\Delta C$.

   b) A survey of 500 televisions viewers of sports channel produced the following information : 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football, 70 watch cricket and hockey, 50 watch hockey and football and 50 do not watch any of the three games.

      i) How many viewers in the survey watch all three kinds of games ?

      ii) How many viewers watch exactly one of the sports ? **(5+5)**

10. a) Draw the Hasse diagram representing the partial ordering { (a, b) / a divides b}.

    b) Let A and B be two non-empty sets. Define :

       i) A function from A to B

       ii) One-to-one function

       iii) On-to function

       iv) Bijective function. If $|A| = 4$ and $|B| = 7$, find the number of functions from A to B and one to one functions from A to B. **(5+5)**

11. a) Prove that, for any propositions p, q and r the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a Tautology.

    b) If a band could not play rock music or the refreshments were not delivered on time, then the new year party would have been cancelled and Alicia would have been angry. If the party were cancelled, then refunds would have to be made. No refunds were made. Therefore, the band could play rock music. Establish the validity of the argument by using the rules of inferences. **(5+5)**

12. a) For all positive integers n, prove that if $n \geq 24$, then n can be written as a sum of 5's and / or 7's.

b) State and prove the extended pigeon hole principle. **(5+5)**

13. a) Three coins are tossed in succession. Find out the probabilities of occurrence of

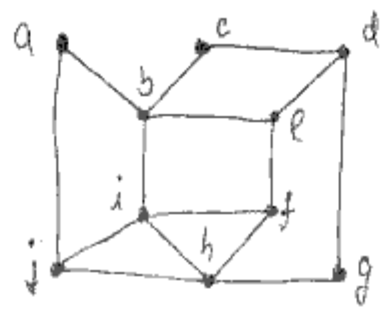
i) Two consecutive heads

ii) Two heads and

iii) Two heads in the following order, head, tail and head.

b) Let $b_0, b_1, b_2, \ldots$ be defined by the formula $b_n = 4^n$, for all integers $n \geq 0$. Show that this sequence satisfies the recurrence relation $b_k = 4b_{k-1}$ for all integers ≥ 1. **(5+5)**

14. a) Prove that the graph G shown below does not have a Hamiltonian circuit. **(5+5)**



b) Find the length of a shortest path between a and z in the weighted graph. **(5+5)**