PG – 263

I Semester M.C.A. Degree Examination, January 2016
(CBCS)
COMPUTER SCIENCE
MCA – 104 T : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **any five** questions from Part – **A** and **any four** from Part – **B**.*

PART – A

1. a) Determine the sets A and B, given that $A - B = \{1, 2, 4\}$, $B - A = \{7, 8\}$ and $A \cup B = \{1, 2, 4, 5, 7, 8, 9\}$.

   b) Prove that, for any three sets A, B and C (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$. **(3+3)**

2. a) Let x be the set of factors of 12 and let $\leq$ be the relation divisor i.e., $x \leq y$ if and only if x divides y. Draw the Hasse diagram of $(X, \leq)$.

   b) $f : Z \to N$ is defined by $f(x) = \begin{cases} 2x - 1, & \text{if } x > 0 \\ -2x, & \text{if } x \leq 0 \end{cases}$. Prove that f is one-to-one and onto. **(3+3)**

3. Prove the validity of the following arguments : **6**

   i) $p \to r$
   $\neg p \to q$
   $q \to s$
   ―――――
   $\therefore \neg r \to s$

   ii) $(\neg p \vee \neg q) \to (r \wedge s)$
   $r \to s$
   $\neg t$
   ―――――
   $\therefore p$

4. Obtain an explicit form for the following sequences $\{a_n\}$ defined recursively by $a_n = 2a_{n-1} + 1$ for $n \geq 2$, with $a_1 = 3$. **6**

5. The probability that an integrated circuit will have defective etching is 0.12, the probability that it will have a crack defect is 0.29, and the probability that it will have both defects is 0.07. What is the probability that a newly manufactured chip will have (i) an etching or crack defect ? (ii) neither defect ? **6**

P.T.O.

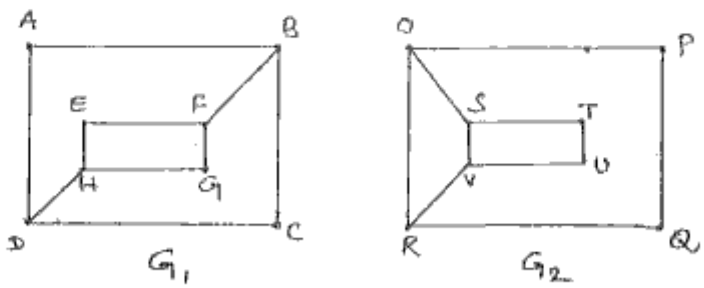
6. Prove the following :

   a) If A and B are countable sets, then $A \cup B$ is countable.

   b) If A and B are countable sets, then $A - B$ and $B - A$ are countable. **(3+3)**

7. Find the number and sum of all positive divisors of 24. **6**

8. Examine whether the following pair of graphs are isomorphic or not. Justify your answer. **6**



PART – B

9. a) Draw a Venn diagram for the following and solve $n(A) = 32$, $n(B) = 29$, $n(A \cap B) = 11$, $n(B \cap C) = 12$, $n(A \cap C) = 13$ and $n(A \cap B \cap C) = 5$. Hence $n(A \cup B \cup C)$, n(only A), n(only B) and n(only C).

   b) Thirty cars are assembled in a factory. The options available are a music system, an air conditioner and power windows. It is known that 15 of the cars have music systems, 8 have air conditioners and 6 have power windows. Further, 3 have all options. Determine atleast how many cars do not have any option at all. **(5+5)**

10. a) If I be the set of all integers and if the relation R be defined over the set I by xRy if $x - y$ is an even integer, where $x, y \in I$, show that R is an equivalence relation.

    b) Consider the functions $f, g : R \to R$, defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 1$. Find the composition function $(g \circ f)(x)$ and $(f \circ g)(x)$. **(5+5)**

11. a) Prove that $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$.

    b) Prove the following : "If $\neg p \leftrightarrow q$ is true, $q \to r$ is true and $\neg r$ is true then p is true" by the method of contradiction. **(5+5)**

12. a) State and prove the pigeon hole principle.

b) For all positive integers n, prove that if $n \geq 24$, then n can be written as a sum of 5's and or 7's. **(5+5)**

13. a) Two cards are drawn from a pack of cards at random. What is the probability that it will be

i) a diamond and a heart

ii) a king and a queen

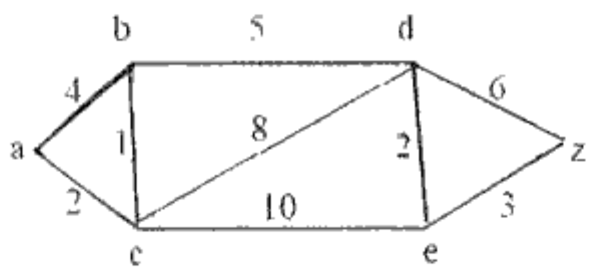
iii) two kings

b) Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 0$, $a_1 = -9$ and $a_2 = 15$. **(5+5)**

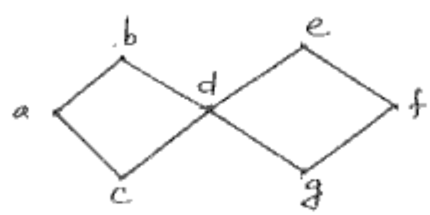
14. a) If G is an Euler graph, show that all the vertices of G are of even degree. **5**

b) Prove that a tree with n vertices has (n – 1) edges. **5**

15. a) Find the length of a shortest path between a to z in the weighted graph.



b) Find the spanning tree of graph G.



(5+5)